UNIVERSITY OF ESWATINI

FIRST SEMESTER MAIN EXAMINATION PAPER, MAY 2021

FACULTY OF SOCIAL SCIENCES

DEPARTMENT OF STATISTICS AND DEMOGRAPHY

COURSE CODE: STA409

TITLE OF PAPER: NON-PARAMETRIC STATISTICS

TIME ALLOWED: 2 HOURS

Instructions

1. Show all your working.
2. Answer any THREE questions.

Special Requirements

Scientific calculator

Additional Material (s)

1. Statistical Tables

Candidates may complete the front cover of their answer book when instructed by the Chief Invigilator and sign their examination attendance cards but must NOT write anything else until the start of the examination period is announced.

No electronic devices capable of storing and retrieving text, including electronic dictionaries and any form of foreign material may be used while in the examination room.

DO NOT turn examination paper over until instructed to do so.

Question 1 [*13+7 marks*]

a. Consider the binomial test, derive the exact distribution of B using the equation

$$B = \sum_{i=1}^{n} \psi_i$$

where

$$\psi_i = \begin{cases} 1, & \text{if the } ith \text{ Bernoulli trial is a success,} \\ 0, & \text{if the } ith \text{ Bernoulli trial is failure.} \end{cases}$$

in the case when $n = 2$ and $p = 0.25$ and hence calculate $P_{0.25}(B \geq 1)$

b. It is estimated that at least half of the men who currently undergo an operation to remove prostrate cancer suffer from a particular undesirable side effect. In an effort to reduce the likelihood of this side effect the FDA studied a new method of performing the operation. Out of 19 operations only 3 men suffered the unpleasant side effect. Is it safe to conclude that the new method of operating is effective in reducing the side effect? Use $\alpha = 0.05$, and p-value approach.

Question 2 [*14+6 marks*]

Let $Z_i = Y_i - X_i$, consider the Wilcoxon signed rank test statistic

$$T^+ = \sum_{i=1}^{n} R_i \psi_i,$$

where ψ_i is the indicator variable

$$\psi_i = \begin{cases} 1, & \text{if } Z_i > 0 \\ 0, & \text{if } Z_i < 0. \end{cases}$$

a. For the case of $n = 3$, derive the distribution of T^+ under H_0 (no ties case).

b. Using your answer from (a) above:

   i. Find $P_0(T^+ \geq 5)$

   ii. Calculate the mean and variance of T^+ under the null hypothesis.

Question 3 [*10+10 marks*]

a. To determine if a new treatment is effective in treating sleep apnea, individual suffering from sleep apnea were enrolled in a study. The number of hours of uninterrupted sleep prior to and after treatment was measured. The data is provided in the following table. Test the appropriate hypothesis at the 5% level of significance, use large sample approximation. Report your conclusions.

Individual	1	2	3	4	5	6	7	8	9	10
Before	6.2	4.4	5.9	6.4	5.8	6.1	6.7	7.0	5.6	4.9
After	7.1	8.0	4.9	7.8	6.2	8.2	5.6	7.2	6.5	5.2

b. In a comparative study of the lifetimes of three different brands of light bulbs, three independent random samples from each brand were tested to see how long the lasted (in hours) with the following results.

Brand	Lifetime (hours)					
1	80.5	82.4	88.9	95.6	102.8	111.7
2	81.7	84.3	86.8	92.3	98.5	101.9
3	98.2	108.7	118.1	124.5	130.9	138.4

Use an appropriate test to show that, at the 10% significance level, there are differences between the brands in terms of median lifetime.

Question 4 [*12+ 8 marks*]

a. A graduate student performed a pilot study for his dissertation. He wanted to examine the effects of animal companionship on elderly males. He selected 10 male participants from a nursing home. Then he used an ABAB research design, where A represented a week with the absence of a cat and B represented a week with the presence of a cat. At the end of each week, he administered a 20-point survey to measure quality of life satisfaction. The survey results are presented in table below.

Participant	Week 1	Week 2	Week 3	Week 4
1	7	6	8	9
2	9	8	10	7
3	15	18	16	17
4	7	6	8	9
5	7	8	10	11
6	10	14	13	11
7	12	19	11	13
8	7	4	2	5
9	8	7	9	5
10	12	16	14	15

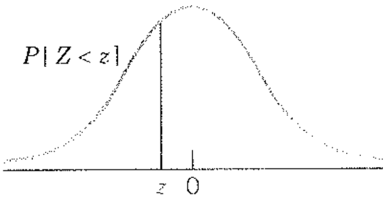
Use an appropriate test to determine if one or more of the groups are significantly different. Since this is pilot study, use $\alpha = 0.10$.

b. The business department at a small college wanted to compare the relative class rank of its MBA graduates with their fifth-year salaries. The data collected by the department are presented in table below. Compare the graduates class rank with their fifth-year salaries

Relative class rank	Fifth year salary
1	83450
2	67900
3	89000
4	80500
5	91000
6	55440
7	101300
8	50560
9	76050

Use a two-tailed Spearman rank-order correlation with $\alpha = 0.05$ to determine if a relationship exists between the two variables.

TABLE 3 Standard Normal Probabilities

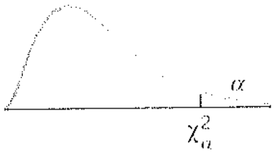


z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−.7	.2420	.2389	.2358	.2327	.2297	.2266	.2236	.2206	.2177	.2148
−.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

TABLE 2 *(Continued)*

		p										
		.05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95
	c											
	6	1.000	.998	.913	.608	.250	.058	.006	.000	.000	.000	.000
	7	1.000	1.000	.968	.772	.416	.132	.021	.001	.000	.000	.000
	8	1.000	1.000	.990	.887	.596	.252	.057	.005	.000	.000	.000
	9	1.000	1.000	.997	.952	.755	.412	.128	.017	.001	.000	.000
	10	1.000	1.000	.999	.983	.872	.588	.245	.048	.003	.000	.000
	11	1.000	1.000	1.000	.995	.943	.748	.404	.113	.010	.000	.000
	12	1.000	1.000	1.000	.999	.979	.868	.584	.228	.032	.000	.000
	13	1.000	1.000	1.000	1.000	.994	.942	.750	.392	.087	.002	.000
	14	1.000	1.000	1.000	1.000	.998	.979	.874	.584	.196	.011	.000
	15	1.000	1.000	1.000	1.000	1.000	.994	.949	.762	.370	.043	.003
	16	1.000	1.000	1.000	1.000	1.000	.999	.984	.893	.589	.133	.016
	17	1.000	1.000	1.000	1.000	1.000	1.000	.996	.965	.794	.323	.075
	18	1.000	1.000	1.000	1.000	1.000	1.000	.999	.992	.931	.608	.264
	19	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	.988	.878	.642
	20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$n = 25$	0	.277	.072	.004	.000	.000	.000	.000	.000	.000	.000	.000
	1	.642	.271	.027	.002	.000	.000	.000	.000	.000	.000	.000
	2	.873	.537	.098	.009	.000	.000	.000	.000	.000	.000	.000
	3	.966	.764	.234	.033	.002	.000	.000	.000	.000	.000	.000
	4	.993	.902	.421	.090	.009	.000	.000	.000	.000	.000	.000
	5	.999	.967	.617	.193	.029	.002	.000	.000	.000	.000	.000
	6	1.000	.991	.780	.341	.074	.007	.000	.000	.000	.000	.000
	7	1.000	.998	.891	.512	.154	.022	.001	.000	.000	.000	.000
	8	1.000	1.000	.953	.677	.274	.054	.004	.000	.000	.000	.000
	9	1.000	1.000	.983	.811	.425	.115	.013	.000	.000	.000	.000
	10	1.000	1.000	.994	.902	.586	.212	.034	.002	.000	.000	.000
	11	1.000	1.000	.998	.956	.732	.345	.078	.006	.000	.000	.000
	12	1.000	1.000	1.000	.983	.846	.500	.154	.017	.000	.000	.000
	13	1.000	1.000	1.000	.994	.922	.655	.268	.044	.002	.000	.000
	14	1.000	1.000	1.000	.998	.966	.788	.414	.098	.006	.000	.000
	15	1.000	1.000	1.000	1.000	.987	.885	.575	.189	.017	.000	.000
	16	1.000	1.000	1.000	1.000	.996	.946	.726	.323	.047	.000	.000
	17	1.000	1.000	1.000	1.000	.999	.978	.846	.488	.109	.002	.000
	18	1.000	1.000	1.000	1.000	1.000	.993	.926	.659	.220	.009	.000
	19	1.000	1.000	1.000	1.000	1.000	.998	.971	.807	.383	.033	.001
	20	1.000	1.000	1.000	1.000	1.000	1.000	.991	.910	.579	.098	.007
	21	1.000	1.000	1.000	1.000	1.000	1.000	.998	.967	.766	.236	.034
	22	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.991	.902	.463	.127
	23	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.998	.973	.729	.358
	24	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.996	.928	.723
	25	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

TABLE 5 Percentage Points of χ^2 Distributions



α d.f.	.99	.975	.95	.90	.50	.10	.05	.025	.01
1	.0002	.001	.004	.02	.45	2.71	3.84	5.02	6.63
2	.02	.05	.10	.21	1.39	4.61	5.99	7.38	9.21
3	.11	.22	.35	.58	2.37	6.25	7.81	9.35	11.34
4	.30	.48	.71	1.06	3.36	7.78	9.49	11.14	13.28
5	.55	.83	1.15	1.61	4.35	9.24	11.07	12.83	15.09
6	.87	1.24	1.64	2.20	5.35	10.64	12.59	14.45	16.81
7	1.24	1.69	2.17	2.83	6.35	12.02	14.07	16.01	18.48
8	1.65	2.18	2.73	3.49	7.34	13.36	15.51	17.53	20.09
9	2.09	2.70	3.33	4.17	8.34	14.68	16.92	19.02	21.67
10	2.56	3.24	3.94	4.87	9.34	15.99	18.31	20.48	23.21
11	3.05	3.81	4.57	5.58	10.34	17.28	19.68	21.92	24.72
12	3.57	4.40	5.23	6.30	11.34	18.55	21.03	23.34	26.22
13	4.11	5.01	5.89	7.04	12.34	19.81	22.36	24.74	27.69
14	4.66	5.62	6.57	7.79	13.34	21.06	23.68	26.12	29.14
15	5.23	6.26	7.26	8.55	14.34	22.31	25.00	27.49	30.58
16	5.81	6.90	7.96	9.31	15.34	23.54	26.30	28.85	32.00
17	6.41	7.56	8.67	10.09	16.34	24.77	27.59	30.19	33.41
18	7.01	8.23	9.39	10.86	17.34	25.99	28.87	31.53	34.81
19	7.63	8.90	10.12	11.65	18.34	27.20	30.14	32.85	36.19
20	8.26	9.59	10.85	12.44	19.34	28.41	31.41	34.17	37.57
21	8.90	10.28	11.59	13.24	20.34	29.62	32.67	35.48	38.93
22	9.54	10.98	12.34	14.04	21.34	30.81	33.92	36.78	40.29
23	10.20	11.69	13.09	14.85	22.34	32.01	35.17	38.08	41.64
24	10.86	12.40	13.85	15.66	23.34	33.20	36.42	39.36	42.98
25	11.52	13.11	14.61	16.47	24.34	34.38	37.65	40.65	44.31
26	12.20	13.84	15.38	17.29	25.34	35.56	38.89	41.92	45.64
27	12.88	14.57	16.15	18.11	26.34	36.74	40.11	43.19	46.96
28	13.56	15.30	16.93	18.94	27.34	37.92	41.34	44.46	48.28
29	14.26	16.04	17.71	19.77	28.34	39.09	42.56	45.72	49.59
30	14.95	16.78	18.49	20.60	29.34	40.26	43.77	46.98	50.89
40	22.16	24.42	26.51	29.05	39.34	51.81	55.76	59.34	63.69
50	29.71	32.35	34.76	37.69	49.33	63.17	67.50	71.42	76.15
60	37.48	40.47	43.19	46.46	59.33	74.40	79.08	83.30	88.38
70	45.44	48.75	51.74	55.33	69.33	85.53	90.53	95.02	100.43
80	53.54	57.15	60.39	64.28	79.33	96.58	101.88	106.63	112.33
90	61.75	65.64	69.13	73.29	89.33	107.57	113.15	118.14	124.12
100	70.06	74.22	77.93	82.36	99.33	118.50	124.34	129.56	135.81

TABLE B.5 Critical Values for the Friedman Test Statistic F_r.

k	N	$\alpha \leq 0.10$	$\alpha \leq 0.05$	$\alpha \leq 0.025$	$\alpha \leq 0.01$
3	3	6.000	6.000		
	4	6.000	6.500	8.000	8.000
	5	5.200	6.400	7.600	8.400
	6	5.333	7.000	8.333	9.000
	7	5.429	7.143	7.714	8.857
	8	5.250	6.250	7.750	9.000
	9	5.556	6.222	8.000	8.667
	10	5.000	6.200	7.800	9.600
	11	4.909	6.545	7.818	9.455
	12	5.167	6.500	8.000	9.500
	13	4.769	6.000	7.538	9.385
	14	5.143	6.143	7.429	9.000
	15	4.933	6.400	7.600	8.933
4	2	6.000	6.000		
	3	6.600	7.400	8.200	9.000
	4	6.300	7.800	8.400	9.600
	5	6.360	7.800	8.760	9.960
	6	6.400	7.600	8.800	10.200
	7	6.429	7.800	9.000	10.371
	8	6.300	7.650	9.000	10.500
	9	6.467	7.800	9.133	10.867
	10	6.360	7.800	9.120	10.800
	11	6.382	7.909	9.327	11.073
	12	6.400	7.900	9.200	11.100
	13	6.415	7.985	7.369	11.123
	14	6.343	7.886	9.343	11.143
	15	6.440	8.040	9.400	11.240
5	2	7.200	7.600	8.000	8.000
	3	7.467	8.533	9.600	10.133
	4	7.600	8.800	9.800	11.200
	5	7.680	8.960	10.240	11.680
	6	7.733	9.067	10.400	11.867
	7	7.771	9.143	10.514	12.114
	8	7.800	9.300	10.600	12.300
	9	7.733	9.244	10.667	12.444
	10	7.760	9.280	10.720	12.480
6	2	8.286	9.143	9.429	9.714
	3	8.714	9.857	10.810	11.762
	4	9.000	10.286	11.429	12.714
	5	9.000	10.486	11.743	13.229
	6	9.048	10.571	12.000	13.619
	7	9.122	10.674	12.061	13.857

(Continued)

TABLE B.5 ***(Continued)***

k	*N*	$\alpha \leq 0.10$	$\alpha \leq 0.05$	$\alpha \leq 0.025$	$\alpha \leq 0.01$
	8	9.143	10.714	12.214	14.000
	9	9.127	10.778	12.302	14.143
	10	9.143	10.800	12.343	14.229

Source: Adapted from Martin, L., Leblanc, R., & Toan, N. K. (1993). Tables for the Friedman rank test. *The Canadian Journal of Statistics / La Revue Canadienne de Statistique, 21*(1), 39–43. Reprinted with permission from *The Canadian Journal of Statistics*. Copyright 1993 by the Statistical Society of Canada. All rights reserved.

TABLE B.6 The Critical Values for the Kruskal–Wallis *H*-Test Statistic.

(The Critical Values for the Kruskal–Wallis *H*-Test Statistic, $k = 3$).

n_1	n_2	n_3	$\alpha \leq 0.10$	$\alpha \leq 0.05$	$\alpha \leq 0.01$
2	2	2	4.571429	–	–
3	1	1	–	–	–
3	2	1	4.285714	–	–
3	2	2	4.464286	4.714286	–
3	3	1	4.571429	5.142857	–
3	3	2	4.555556	5.361111	–
3	3	3	4.622222	5.600000	6.488889
4	2	1	4.500000	–	–
4	2	2	4.458333	5.333333	–
4	3	1	4.055556	5.208333	–
4	3	2	4.511111	5.444444	6.444444
4	3	3	4.700000	5.790909	6.745455
4	4	1	4.166667	4.966667	6.666667
4	4	2	4.554545	5.454545	7.036364
4	4	3	4.515455	5.598485	7.143939
4	4	4	4.653846	5.692308	7.653846
5	2	1	4.200000	5.000000	–
5	2	2	4.373333	5.160000	6.533333
5	3	1	4.017778	4.871111	–
5	3	2	4.650909	5.250909	6.821818
5	3	3	4.533333	5.648485	7.078788
5	4	1	3.987273	4.985455	6.954545
5	4	2	4.540909	5.272727	7.204545
5	4	3	4.548718	5.656410	7.444872
5	4	4	4.668132	5.657143	7.760440
5	5	1	4.109091	5.127273	7.309091
5	5	2	4.623077	5.338462	7.338462
5	5	3	4.545055	5.626374	7.578022
5	5	4	4.522857	5.665714	7.791429
5	5	5	4.560000	5.780000	8.000000

TABLE B.6 (***Continued***)

(The Critical Values for the Kruskal–Wallis H-Test Statistic, $k = 3$).

n_1	n_2	n_3	$\alpha \leq 0.10$	$\alpha \leq 0.05$	$\alpha \leq 0.01$
6	2	1	4.200000	4.822222	–
6	2	2	4.436364	5.345455	6.654545
6	3	1	3.909091	4.854545	6.581818
6	3	2	4.681818	5.348485	6.969697
6	3	3	4.538462	5.615385	7.192308
6	4	1	4.037879	4.946970	7.083333
6	4	2	4.493590	5.262821	7.339744
6	4	3	4.604396	5.604396	7.467033
6	4	4	4.523810	5.666667	7.795238
6	5	1	4.128205	4.989744	7.182051
6	5	2	4.595604	5.318681	7.375824
6	5	3	4.535238	5.601905	7.590476
6	5	4	4.522500	5.660833	7.935833
6	5	5	4.547059	5.698529	8.027941
6	6	1	4.000000	4.857143	7.065934
6	6	2	4.438095	5.409524	7.466667
6	6	3	4.558333	5.625000	7.725000
6	6	4	4.547794	5.724265	8.000000
6	6	5	4.542484	5.764706	8.118954
6	6	6	4.538012	5.719298	8.222222
7	1	1	4.266667	–	–
7	2	1	4.200000	4.706494	–
7	2	2	4.525974	5.142857	7.000000
7	3	1	4.173160	4.952381	6.649351
7	3	2	4.582418	5.357143	6.838828
7	3	3	4.602826	5.620094	7.227630
7	4	1	4.120879	4.986264	6.986264
7	4	2	4.549451	5.375981	7.304553
7	4	3	4.527211	5.623129	7.498639
7	4	4	4.562500	5.650000	7.814286
7	5	1	4.035165	5.063736	7.060597
7	5	2	4.484898	5.392653	7.449796
7	5	3	4.535238	5.588571	7.697143
7	5	4	4.541597	5.732773	7.931092
7	5	5	4.540056	5.707563	8.100840
7	6	1	4.032653	5.066667	7.254422
7	6	2	4.500000	5.357143	7.490476
7	6	3	4.550420	5.672269	7.756303
7	6	4	4.561625	5.705882	8.016340
7	6	5	4.559733	5.769925	8.156725

(Continued)

TABLE B.6 (*Continued*)

(The Critical Values for the Kruskal–Wallis H-Test Statistic, $k = 4$.)

n_1	n_2	n_3	n_4	$\alpha \leq 0.10$	$\alpha \leq 0.05$	$\alpha \leq 0.01$
5	5	4	1	5.670000	6.782500	8.870000
5	5	4	2	5.944853	7.032353	9.156618
5	5	4	3	6.052288	7.217647	9.356863
5	5	4	4	6.070175	7.291228	9.536842
5	5	5	1	5.682353	6.829412	9.052941
5	5	5	2	5.945098	7.074510	9.286275
5	5	5	3	6.043275	7.250292	9.495906
5	5	5	4	6.082105	7.327895	9.669474
5	5	5	5	6.097143	7.377143	9.800000

Source: Adapted from Meyer, J. P., & Seaman, M. A. (2008, March). A comparison of the exact Kruskal-Wallis distribution to asymptotic approximations for $N \leq 105$. Paper presented at the annual meeting of the American Educational Research Association, New York. Reprinted with permission of the authors.

TABLE B.7 Critical Values for the Spearman Rank-Order Correlation Coefficient r_s.

n	$\alpha_{\text{two-tailed}} \leq 0.10$ $\alpha_{\text{one-tailed}} \leq 0.05$	$\alpha_{\text{two-tailed}} \leq 0.05$ $\alpha_{\text{one-tailed}} \leq 0.025$	$\alpha_{\text{two-tailed}} \leq 0.02$ $\alpha_{\text{one-tailed}} \leq 0.01$	$\alpha_{\text{two-tailed}} \leq 0.01$ $\alpha_{\text{one-tailed}} \leq 0.005$
4	1.000			
5	0.900	1.000	1.000	
6	0.829	0.886	0.943	1.000
7	0.714	0.786	0.893	0.929
8	0.643	0.738	0.833	0.881
9	0.600	0.700	0.783	0.833
10	0.564	0.648	0.745	0.794
11	0.536	0.618	0.709	0.755
12	0.503	0.587	0.671	0.727
13	0.484	0.560	0.648	0.703
14	0.464	0.538	0.622	0.675
15	0.443	0.521	0.604	0.654
16	0.429	0.503	0.582	0.635
17	0.414	0.485	0.566	0.615
18	0.401	0.472	0.550	0.600
19	0.391	0.460	0.535	0.584
20	0.380	0.447	0.520	0.570
21	0.370	0.435	0.508	0.556
22	0.361	0.425	0.496	0.544
23	0.353	0.415	0.486	0.532

(*Continued*)

TABLE B.7 ***(Continued)***

n	$\alpha_{\text{two-tailed}} \leq 0.10$ $\alpha_{\text{one-tailed}} \leq 0.05$	$\alpha_{\text{two-tailed}} \leq 0.05$ $\alpha_{\text{one-tailed}} \leq 0.025$	$\alpha_{\text{two-tailed}} \leq 0.02$ $\alpha_{\text{one-tailed}} \leq 0.01$	$\alpha_{\text{two-tailed}} \leq 0.01$ $\alpha_{\text{one-tailed}} \leq 0.005$
24	0.344	0.406	0.476	0.321
25	0.337	0.398	0.466	0.511
26	0.331	0.390	0.457	0.501
27	0.324	0.382	0.448	0.491
28	0.317	0.375	0.440	0.483
29	0.312	0.368	0.433	0.475
30	0.306	0.362	0.425	0.467
31	0.301	0.356	0.418	0.459
32	0.296	0.350	0.412	0.452
33	0.291	0.345	0.405	0.446
34	0.287	0.340	0.399	0.439
35	0.283	0.335	0.394	0.433
36	0.279	0.330	0.388	0.427
37	0.275	0.325	0.383	0.421
38	0.271	0.321	0.378	0.415
39	0.267	0.317	0.373	0.410
40	0.264	0.313	0.368	0.405
41	0.261	0.309	0.364	0.400
42	0.257	0.305	0.359	0.395
43	0.254	0.301	0.355	0.391
44	0.251	0.298	0.351	0.386
45	0.248	0.294	0.347	0.382
46	0.246	0.291	0.343	0.378
47	0.243	0.288	0.340	0.374
48	0.240	0.285	0.336	0.370
49	0.238	0.282	0.333	0.366
50	0.235	0.279	0.329	0.363

Source: Adapted from Zar, J. H. (1972). Significance testing of the Spearman rank correlation coefficient. *Journal of the American Statistical Association*, *67*, 578–580. Reprinted with permission from The *Journal of the American Statistical Association*. Copyright 1972 by the American Statistical Association. All rights reserved.

TABLE B.10 Critical Values for the Runs Test for Randomness.

One-tailed alternative; $\alpha = 0.05$.

	n_2										
n_1	2	3	4	5	6	7	8	9	10	11	12
2	–	–	–	–	–	–	2	2	2	2	2
	–	–	–	–	–	–	–	–	–	–	–
3	–	–	–	2	2	2	2	2	3	3	3
	–	–	7	–	–	–	–	–	–	–	–
4	–	–	2	2	3	3	3	3	3	3	4
	–	7	8	9	9	9	–	–	–	–	–
5	–	2	2	3	3	3	3	4	4	4	4
	–	–	9	9	10	10	11	11	11	–	–
6	–	2	3	3	3	4	4	4	5	5	5
	–	–	9	10	11	11	12	12	12	13	13
7	–	2	3	3	4	4	4	5	5	5	6
	–	–	9	10	11	12	13	13	13	14	14
8	2	2	3	3	4	4	5	5	6	6	6
	–	–	–	11	12	13	13	14	14	15	15
9	2	2	3	4	4	5	5	6	6	6	7
	–	–	–	11	12	13	14	14	15	15	16
10	2	3	3	4	5	5	6	6	6	7	7
	–	–	–	11	12	13	14	15	16	16	17
11	2	3	3	4	5	5	6	6	7	7	8
	–	–	–	–	13	14	15	15	16	17	17
12	2	3	4	4	5	6	6	7	7	8	8
	–	–	–	–	13	14	15	16	17	17	18

One-tailed alternative; $\alpha = 0.025$.

	n_2										
n_1	2	3	4	5	6	7	8	9	10	11	12
2	–	–	–	–	–	–	–	–	–	–	2
	–	–	–	–	–	–	–	–	–	–	–
3	–	–	–	–	2	2	2	2	2	2	2
	–	–	–	–	–	–	–	–	–	–	–
4	–	–	–	2	2	2	3	3	3	3	3
	–	–	–	9	9	–	–	–	–	–	–
5	–	–	2	2	3	3	3	3	3	4	4
	–	–	9	10	10	11	11	–	–	–	–
6	–	2	2	3	3	3	3	4	4	4	4
	–	–	9	10	11	12	12	13	13	13	13
7	–	2	2	3	3	3	4	4	5	5	5
	–	–	–	11	12	13	13	14	14	14	14

(Continued)